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Development of data normalization methods for multi-criteria decision making: applying for MARCOS method

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Abstract. The purpose of the data normalization is to transfer the quantities with different dimensions to the same dimensionless form. The multi-criteria decision-making (*MCDM*) methods that require identifying the weight for each criterion, so the data normalization should be performed. In this study, five distinct data normalization methods were used in combination with a multi-criteria decision-making method (*MARCOS* method). All five of these data normalization methods were performed in combining with the *MARCOS* method and applied in three different cases. The number of solutions and the criteria in each case were different. Two different weighting methods were also used in each situation. After defining the most suitable data normalization methods in combining with the *MARCOS* method, this study proposed two new data normalization methods. The results show that solution rank is likely stable. The works in the future were mentioned in the last section of this article as well.

Keywords: *MCDM* / data normalization / *MARCOS*

1 Introduction

The multi-criteria decision-making methods identify an alternative that is considered the best among the implemented alternatives. However, many alternatives that are determined to be the best may be not feasible. For instance, a restaurant is considered the best (in terms of food, location, price, etc.), yet that restaurant is closed for some reason. At this point, it is clear that a selected restaurant can be ranked second, even third. Nonetheless, studies on the multi-criteria decision-making so far have often only focused on determining the best option, while the second or third alternatives seem to be neglected. Thus, besides finding the best solution, it is essential to pay attention to the second and third options when making the multi-criteria decisions. This promotes a more comprehensive study upon ranking alternatives and assessing the stability of that ranking result, firstly for the best, second and third best alternative.

At present, there are two *MCDM* groups, one of them requires determining the weights for criteria (group A) and the other does not require the weights for criteria (group B). The methods in group A include: *TOPSIS*, *VIKOR*, *MOORA*, *COPRAS*, *PIV*, *MARCOS*, *RIM*, *WASPAS*, etc. The methods in group B consist of: *PSI*, *PEG*, *CURLI*. It can be said that the number of *MCDM* methods of group A is much larger than of group B. Normalizing the data is needed to carry out upon using the *MCDM* methods of group A. It converts the quantities (criteria) to dimensionless form, then the alternatives can be compared [1,2]. The data normalization also creates an opportunity for decision makers to weigh the criteria (priority) [3]. For example, two criteria for evaluating a machining process are processing productivity and product expense, where the unit of productivity is the number of products produced in an amount of time, and the expense is expressed in monetary units. Normalizing turns these two criteria to a dimensionless form. Each multi-criteria decision-making method mentions the data normalization method itself [3]. However, the different multi-criteria decision making methods using the different data normalization approaches lead to the different rank results [4–6]. In the next section of this research, some of these cases are discussed in more detail. Furthermore, choosing the inappropriate data normalization methods also cause rank reversal problems or inaccurate rank of the alternatives, that means the worst-case scenario or the second worst are ranked as the preferred alternative, causing the best alternative to be missed [3,7,8]. Consequently, the best option to be found might not be the true best alternative during making multi-criteria decisions if only one method of data normalization is applied. This is probably solved if the multiple data normalization methods are used simultaneously for a single decision to be made.

In this study, the *MARCOS* method was combined with some data normalization methods. This *MARCOS* method was chosen because it has outstanding advantages that have been confirmed in many previous studies. The third section of this paper will discuss those advantages. The

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main purpose of this study is determination of the suitable data normalization methods when combining with the *MARCOS* method for multi-criteria decision making. To achieve the set objectives, the main presented contents in the following sections of this study include: (a) data normalization methods commonly used in combination with *MCDM* methods, as well as present some limitations of such data normalization methods; (b) discuss the advantages of the *MARCOS* method through the analysis of the published studies; (c) combine the *MARCOS* and data normalization methods for multi-criteria decision making in several machining processes; (d) propose new methods of data normalization to combine with the *MARCOS* method; (e) discuss the above-mentioned combinations, draw the conclusions and propose the research directions for future research.

2 Methods of data normalization

As mentioned above, normalizing data is transfering the data to the dimensionless form. The data normalization methods have many distinct types. Five data normalization methods that have been used internally in multi-criteria decision-making methods are listed below. The concept of "internally" is understood as the normalizing method used in the multi-criteria decision-making method itself, applied by the inventors of the decision-making methods.

Method I ($N1$)

$$n_{ij}^{(1C)} = \frac{\min x_{ij}}{x_{ij}} \quad \text{if} \quad j \in C \tag{1}$$

$$n_{ij}^{(1B)} = \frac{x_{ij}}{\max x_{ij}} \quad \text{if} \quad j \in B \tag{2}$$

Method II ($N2$)

$$n_{ij}^{(2C)} = \frac{x_{ij} - \max x_{ij}}{\min x_{ij} - \max x_{ij}} \quad \text{if} \quad j \in C \tag{3}$$

$$n_{ij}^{(2B)} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \quad \text{if} \quad j \in B \tag{4}$$

Method III ($N3$)

$$n_{ij}^{(3C)} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad \text{if} \quad j \in C \tag{5}$$

$$n_{ij}^{(3B)} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \quad \text{if} \quad j \in B \tag{6}$$

Method IV ($N4$)

$$n_{ij}^{(4C)} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^{m} \frac{1}{x_{ij}}} \quad \text{if} \quad j \in C \tag{7}$$

$$n_{ij}^{(4B)} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \quad \text{if} \quad j \in B \tag{8}$$

Method V ($N5$)

$$n_{ij}^{(5C)} = \frac{\frac{1-\ln(x_{ij})}{ln(\Pi_{i=1}^{m} x_{ij})}}{m-1} \quad \text{if} \quad j \in C \tag{9}$$

$$n_{ij}^{(5B)} = \frac{\ln(x_{ij})}{ln(\Pi_{i=1}^{m} x_{ij})} \quad \text{if} \quad j \in B. \tag{10}$$

In which, with the formulas from (1) to (10): C represents the min criterion, B represents the max criterion, i is the number of alternatives, j is the number of criterion, and x_{ij} is the value of criterion j at the alternative i.

Table 1 introduces several multi-criteria decision making methods along with the data normalization method used internally.

Table 1 reveals the $N1$ method that is most used internally among the multi-criteria decision making methods. In fact, however, there might be many cases where this method ($N1$) is not able to perform, namely if one of the criteria has $x_{ij} = 0$, then expression (1) will be meaningless. Formula (2) will be meaningless either if max $(x_{ij}) = 0$. Similarly, if there is a value of $x_{ij} = 0$, the $N4$ method cannot be used. The $N5$ method will also be unusable if $x_{ij} \leq 0$. In such cases, the decision makers have few options of the multi-criteria decision-making methods if they do not choose another method of data normalization. However, even if the decision makers choose a different method of data normalization, it raises skepticism about the outcome of the decision. That skepticism is understood whether using a different data normalization method results in the appropriate rank of the alternatives. In order to solve this problem, a multi-criteria decision must firstly be made as the various methods of data normalization are considered. From this point of view, several studies have been conducted on several multi-criteria decision-making methods.

Vafaei et al. [1] used *AHP* (Analytical Hierarchy Process) as a multi-criteria decision-making method for choosing the best smart parking lot. In which they used all five normalizing-data methods mentioned. The research indicates that the $N4$ method cannot be combined with *AHP*, the combination of *AHP* and $N1$ method provided the best results. In contrast, the combination of *AHP* and $N5$ methods provided the worst results.

Vafaei et al. [5] simultaneously used the above five data normalization methods in a decision problem for choosing the landing method of unmanned aircraft. In that study, they applied *TOPSIS* as the decision-making method and claimed that the $N3$ method was the best, while the $N4$ method provided the worst results.

All five data normalization methods mentioned were used by Ersoy in a multi-criteria decision problem as well [6]. His aim was to rank the performance of several companies. The decision-making method in this study was

Table 1. Data normalization methods used internally in different MCDM methods.

Multi Attribute Decision Making	*MCDM*	Normalization method				
		N1	*N2*	*N3*	*N4*	*N5*
Technique for Order Preference by Similarity to Ideal Solution	*TOPSIS*			×		
Vlsekriterijumska optimizacijaI KOmpromisno Resenje (in Serbian)	*VIKOR*		×			
Multiobjective Optimization On the basis of Ratio Analysis	*MOORA*	×				
COmplex PRroportional ASsessment	*COPRAS*	×				
COmbined COmpromise SOlution	*COCOSO*		×			
Simple Additive Weighting	*SAW*	×				
Weighted Aggregates Sum Product ASsessment	*WASPAS*	×				
Proximity Indexed Value	*PIV*	×				
Preference Selection Inde	*PSI*	×				
Multi-Attributive Border Approximation area Comparison	*MABACH*		×			
Preference Analysis for Reference Ideal Solution	*PARIS*	×	×	×		
Mixed Aggregation by COmprehensive Normalization Technique	*MACONT*	×	×			
Weighted Product Model	*WPM*					×
Weighted Sum Model	*WSM*				×	
Range Of Value	*ROV*		×			
Measurement Alternatives and Ranking according to COmpromise Solution	*MARCOS*	×				

the *ROV* method. These research results pointed out that the combination of *ROV* with *N*1 method is the best, while the combination of ROV and *N*4 method should be avoided.

Palczewski et al. [9] used the above five data normalization methods above in the multi-criteria decision making for airport construction. *PROMETHEE II* was chosen as the multi-criteria decision-making method. They concluded that the rank order of alternatives was very different based on the different data normalization methods.

Lakshmi et al. [10] used the *TOPSIS* method to make a multi-criteria decision for car selection. All the five data normalization methods mentioned above were also used. They claimed that using the *N*3 method provided the best results.

Aytekin [3] used the *SAW* method for a randomly designed dataset in the multi-criteria decision making, where he also applied several different data normalization methods. His research indicated that the rank order of alternatives is highly dependent on the data normalization method. Among the data normalization methods used, there is only one best method.

Some authors [11] applied five multi-criteria decision making methods including *AHP*, *Fuzzy AH*P, *TOPSIS*, *Fuzzy TOPSIS* and *PROMETHE*E. All five of these methods are combined with the *N*3 method. The study showed that the result of ranking the alternatives was not the same in those five combinations.

Some above published studies demonstrate that: (1) For each different multi-criteria decision-making method, each data normalization method does not lead to the same results, that means, each decision-making method is suitable for only one or several data normalization methods; (2) The mix of the data normalization methods and many different multi-criteria decision-making methods results in different ranks as well. Thus, in order to choose the best method of normalizing data combined with a certain multi-criteria decision-making method, it is necessary to first use multiple normalization methods simultaneously.

Furthermore, each different method of data normalization gives different normalized values. To demonstrate this statement, a random example is considered as follows: There are three alternatives *A*1, *A*2 and *A*3. Each alternative is evaluated using two criteria including y_1 and y_2. Where y_1 is a type-*C* criterion, and y_2 is a type-*B* criterion (Tab. 2).

Formulas (1) and (2) are used to normalize the data according to *N*1 method; formulas (3) and (4) are applied for normalizing the data based on the *N*2 method; data normalization according to the *N*3 method is carried out using formulas (5) and (6) and ; formulas (7) and (8) are used to normalize the data on the basis of the *N*4 method; the data normalized applying the *N*5 method is determined through two formulas (9) and (10). The normalized results are presented in Table 3.

The data in Table 3 revealed that each method of data normalization gave the different normalized values. In addition, after normalizing the data, the normalized values are multiplied by the weight of the criteria to perform further operations. It is clear that the calculations are also different using the distinct data normalization methods. Furthermore, the use of more than one method of determining the weights for the criteria contributes to the wide variation of the results. This may lead to the different ranks of the alternatives. These prompted a study that needed to consider multiple methods of data normalization as well as multiple methods of determining weights in each particular case.

Table 2. Illustrative data for normalizing by different methods.

Alternatives	y_1	y_2
A_1	5	6
A_2	7	4
A_3	12	8

Table 3. Data normalization results by different methods.

Alternatives	y_1	y_2	Normalization method									
			*N*1		*N*2		*N*3		*N*4		*N*5	
A_1	5	6	1.0000	0.7500	1.0000	0.5000	0.6614	0.5571	0.4693	0.3333	–0.0504	0.3408
A_2	7	4	0.7143	0.5000	0.7143	0.0000	0.5259	0.3714	0.3352	0.2222	–0.0783	0.2637
A_3	12	8	0.4167	1.0000	0.0000	1.0000	0.1873	0.7428	0.1955	0.4444	–0.1229	0.3955

In this paper, *MARCOS* was chosen as the multi-criteria decision-making method. The reason is due to the fact that this method is recently proposed (2020), with high stability upon ranking the alternatives [12], and ability of determining the best solution regardless of the number of alternatives as well as the weighting method used [13].

Two methods of determining the weights including Entropy and *MEREC* (Method based on the Removal Effects of Criteria) are also used simultaneously. They have high accuracy and are specifically recommended to use [14,15].

3 MARCOS method

As mentioned, the multi-criteria decision making based on the *MARCO*S method is able to identify the best alternative regardless of the number of alternatives and the weighting method [13]. Its another advantage is that the rank results are fairly stable [12]. This method was used to rank four project management software, each software was evaluated by six criteria, the weight of the criteria was selected according to seven different value sets. The obtained results showed that when using seven different weight sets, six of them determine the same best software [16]. This method has also been successfully used to evaluate the quality of electronic services in the aviation. Even when, a certain solution was removed from the list, the phenomenon of ranking reversal of the solutions did not occur [17]. In another study about aviation, when using *MARCOS* method to select aircraft for the domestic flights in Turkey, some outstanding advantages of the *MARCOS* method were also found such as determining the best solution with high consistency when using many different weight sets; the best solution that was determined by the *MARCOS* method is similar to that one when obtained using the other seven methods (*MAIRCA*, *WASPAS*, *MOORA*, *SAW*, *CODAS*, *EDAS*, *MABAC*); and the problem of rank inversion also did not occur when a certain solution is removed from the list of the solutions [18]. When using the *MARCOS* method to rank the effective of the logistics performance of five Western Balkan Countries (Bosnia and Herzegovina, North Macedonia, Albania, Serbia, and Montenegro), it consistently determined the best solution despite testing with thirty-six different values of the weight sets of the criteria [19]. In another application of the *MARCOS* method in the transport operations, when comparing the effective of the railway system performance in the Sub-Saharan African region, the best solution that was determined by the *MARCOS* method is similar to that one when using seven other methods (*WASPAS*, *ARAS*, *COCOSO*, *EDAS*, *MABAC*, *SAW*, *TOPSIS*) [20]. In another study, when using the MARCOS method to select the type of forklift truck for the transportation operations, the best solution that was determined using the *MARCOS* method is similar to the that one when using other methods (*WASPAS*, *ARAS*, *COCOSO*, *EDAS*, *MABAC*, *SAW*, *TOPSIS*) [21]. The *MARCOS* method was also successfully applied in selecting the refractory material suppliers in the iron and steel industry in India, and it was also recommended for multi-criteria decision making in in other areas such as defining the maintenance strategy, workshop layout, inventory control policy, and so on [22]. According to the results of the preceding analysis, the MARCOS method has some notable advantages, such as: the ranking results being less dependent on the method of determining the weights for the items; furthermore, when an option was removed from the list of alternatives using this method, no ranking reversal was observed; and it has had a lot of success ranking alternatives in a variety of fields.

Other multi-criteria decision-making methods that have been widely used include *VIKOR*, *MAIRCA*, *CODAS*, *MABAC*, and so on. However, some limitations of these methods have been discovered. Rank inversion often occurs when using the *VIKOR* method [23,24]. When using the *MAIRCA* method, the decision maker must prioritize the alternatives [25]. In this case, the ranking results of the alternatives are frequently influenced by the decision maker's opinions. When using the *CODAS* method, the decision maker must select a threshold to ensure that the Euclidean distance between two alternatives is equal. This threshold is then compared to another set by the decision maker (usually between 0.01 and 0.05). Because of this choice, the ranking result will be heavily influenced by the decision maker's opinion. Another limitation of the *CODAS* method is that when comparing solutions using the Euclidean distance is not possible, the solutions must be compared using the Taxicab distance. This will also create additional difficulties for decision makers [26]. The ranking results of alternatives when using the *MABAC* method are heavily influenced by the weighted values of the criteria. An example of this problem can be found in the rating of flood protection solutions in the city of Arilje in the Republic of Serbia [27]. In this study, the authors proposed four flood protection alternatives that were each evaluated through six criteria, and eight different sets of weights were assigned to the criteria. The results show that when using eight different weighting methods, the two methods determine the same best solution. However, the remaining six methods discovered other one best solution, and the one that ranked once when one weight method was used only ranked three when another method was used. This can be considered the *MABAC* method's weakness in comparison to the *MARCOS* method.

Based on the results of the preceding analysis, the *MARCOS* method was selected for use in this study. However, in the published works using the *MARCOS* method, only one method of data normalization is used (the *N*1 method – Tab. 1). This raises an issue that whether the *MARCOS* method is still advantageous if the multiple data normalization methods are applied. This research aims to address it.

The steps for implementation of multi-criteria decision making according to the *MARCOS* method are as follows [12]:

Step 1: Building an initial matrix based on the following formula.

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \tag{11}$$

where m is the number of alternatives, n is the number of criteria, x_{mn} is the value of criterion n at alternative m.

Step 2: Building an extended initial matrix by adding an ideal solution (AI) and the opposite solution to the ideal solution (AAI).

$$X = \begin{matrix} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{matrix} \begin{bmatrix} x_{aa1} & \cdots & x_{aan} \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ x_{ai1} & \cdots & x_{ain} \end{bmatrix} \tag{12}$$

where

$$AAI = \min(x_{ij}); \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n; \quad \text{if} \quad j \in B$$

$$AAI = \max(x_{ij}); \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n; \quad \text{if} \quad j \in C$$

$$AI = \max(x_{ij}); \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n; \quad \text{if} \quad j \in B$$

$$AI = \min(x_{ij}); \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n; \quad \text{if} \quad j \in C$$

Step 3: Normalizing the extended initial matrix according to the formula.

$$n_{ij} = \frac{x_{AI}}{x_{ij}} \quad \text{if} \quad j \in C \tag{13}$$

$$n_{ij} = \frac{x_{ij}}{x_{AI}} \quad \text{if} \quad j \in B. \tag{14}$$

Note that (13) and (14) are data normalization formulas in the *MARCOS* method itself. This study uses not only these two formulas (the *N*1 method), but also all the normalization methods mentioned above (including *N*2, *N*3, *N*4, *N*5).

Step 4: Defining the normalized value, taking into account the weight of the criteria according to the following formula.

$$v_{ij} = n_{ij} \cdot w_j \tag{15}$$

where w_j is the weight of the criterion j.

Step 5: Calculating the coefficients K_i^+ and K_i^- according to the formula.

$$K_i^- = \frac{S_i}{S_{AAI}} \tag{16}$$

$$K_i^+ = \frac{S_i}{S_{AI}} \tag{17}$$

where S_i, S_{AAI} and S_{AI} are the sum of the values of v_{ij}, x_{aai} and x_{ai}, respectively, where $i = 1, 2, \ldots, m$.

Step 6: Calculating $f(K_i^+)$ and $f(K_i^-)$ according to the formula.

$$f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^i} \tag{18}$$

$$f(K_i^+) = \frac{K_i^-}{K_i^+ + K_i^i} \tag{19}$$

Table 4. Experimental data of EN19 steel turning process [28].

Trial.	Input parameter			Response		
	n (rev/min)	f (mm/rev)	a_p (mm)	Ra (µm)	Rz (µm)	MRR (cm^3/min)
A_1	75	0.05	0.2	2.6	12.6	0.75
A_2	75	0.1	0.4	3.1	14.2	3
A_3	75	0.15	0.6	3.7	15.3	6.75
A_4	150	0.05	0.4	1.8	6.4	3
A_5	150	0.1	0.6	2.3	9.8	9
A_6	150	0.15	0.2	2.8	12.8	4.5
A_7	225	0.05	0.6	0.9	4.1	6.75
A_8	225	0.1	0.2	1.6	7.6	4.5
A_9	225	0.15	0.4	2.1	9.7	13.5

Step 7: Calculating $f(K_i)$ based on (20) and ranking according to the rule that the option with the highest value of the $f(K_i)$ is considered the best.

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1-f\left(K_i^+\right)}{f\left(K_i^+\right)} + \frac{1-f\left(K_i^-\right)}{f\left(K_i^-\right)}}. \quad (20)$$

To evaluate the rank results of alternatives using the five different data normalization methods simultaneously, some examples are performed as below. In the scope of this study, the examples for the multi-criteria decision making are based on machining methods including turning, milling, and grinding. With the main purpose of assessing the ranked results of solutions using the five different data normalization methods as well as many different weighting methods simultaneously, the data of machining operations (turning, milling, and grinding) is used in published works. This also creates a basis for comparing the ranked results of the solutions in this study with those of published research.

4 Examples

4.1 Multi-criteria decision making of turning operation

Experimental data of the EN19 steel turning process were used in a study [28]. In that study, nine experiments were conducted, the spindle speed (n), feed rate (f) and depth of cut (a_p) were changed at each experiment. Three output parameters were also determined including Arithmetic average roughness height (Ra), Ten-point mean roughness (Rz), and Material removal rate (MRR). The result is presented in Table 4. In this study, the authors used three methods including *WSM*, *WPM,* and *TOPSIS* to make multi-criteria decisions about the turning process. The purpose of multi-criteria decision making is to identify one of nine experiments that simultaneously obtain minimum Ra, minimum Rz, and maximum MRR.

This study implemented multi-criteria decision making with the above purpose. The decision-making method was the *MARCOS* method. All the five data normalization methods described in Section 2 were used. Two weighting methods consisting of Entropy and *MEREC* were applied. Details of the steps of determining the weights in the Entropy method are presented in many documents [15,29]. Similarly, the steps for defining the weights of criteria in the *MEREC* method can also be found in some documents [14,30]. Applying the Entropy method identified the weights of Ra, Rz, and MRR that are 0.1932, 0.1998, and 0.6070 respectively. The weights of Ra, Rz, and MRR using the *MEREC* method are 0.1722, 0.1464, and 0.6814, respectively. Firstly, the ranking of alternatives is implemented after the weight of the criteria is determined by the Entropy method, and the data normalization is performed according to the $N1$ method.

The initial matrix is built according to formula (11). It consists of the last three columns in Table 4.

The extended initial matrix is developed according to formula (12), with the results as shown in Table 5.

The results of data normalization by the $N1$ method (formula (13), (14)) are presented in Table 6.

The normalized values taking into account the weights of the criteria is determined by formula (15), with the results as shown in Table 7.

K_i^-, K_i^+, $f(K_i^+)$, $f(K_i^-)$ and $f(K_i)$ are determined according to the respective formulas (16)–(20) and presented in Table 8. The ranked results of the solutions of $f(K_i)$ are included in this table as well.

With a similar implementation, the ranking of solutions after the normalization performed by all five methods and after criteria weights determined by both methods (Entropy and *MEREC*) is conducted. The result is presented in Table 9. The rank results of the alternatives under three approaches including *WSM*, *WPM,* and *TOPSIS* are shown in this table [28].

The data in Table 9 revealed that:

– In the case of the $N1$, $N2$ and $N3$ data normalization method application, the best and worst solutions were the same when the two different weighting methods were used. This is explained by the fact that the *MARCOS* method itself takes into account the ideal (AI) and anti-ideal alternatives (AAI) [13]. In particular, the best and worst solutions found were independent of the data normalization method used. This also coincides with the multi-criteria decision making using the *WSM*, *WPM* and *TOPSIS* methods. Furthermore, the second rank

Table 5. Extended initial matrix.

Trial.	*Ra* (μm)	*Rz* (μm)	*MRR* (cm^3/min)
AAI	3.7	15.3	0.75
A_1	2.6	12.6	0.75
A_2	3.1	14.2	3
A_3	3.7	15.3	6.75
A_4	1.8	6.4	3
A_5	2.3	9.8	9
A_6	2.8	12.8	4.5
A_7	0.9	4.1	6.75
A_8	1.6	7.6	4.5
A_9	2.1	9.7	13.5
AI	0.9	4.1	13.5

Table 6. Normalized values of criteria.

Trial.	*Ra*	*Rz*	*MRR*
AAI	0.2432	0.2680	0.0556
A_1	0.3462	0.3254	0.0556
A_2	0.2903	0.2887	0.2222
A_3	0.2432	0.2680	0.5000
A_4	0.5000	0.6406	0.2222
A_5	0.3913	0.4184	0.6667
A_6	0.3214	0.3203	0.3333
A_7	1.0000	1.0000	0.5000
A_8	0.5625	0.5395	0.3333
A_9	0.4286	0.4227	1.0000
AI	1.0000	1.0000	1.0000

Table 7. Normalized values taking into account criteria weights.

Trial.	*Ra*	*Rz*	*MRR*
AAI	0.0470	0.0535	0.0337
A_1	0.0669	0.0650	0.0337
A_2	0.0561	0.0577	0.1349
A_3	0.0470	0.0535	0.3035
A_4	0.0966	0.1280	0.1349
A_5	0.0756	0.0836	0.4047
A_6	0.0621	0.0640	0.2023
A_7	0.1932	0.1998	0.3035
A_8	0.1087	0.1078	0.2023
A_9	0.0828	0.0845	0.6070
AI	0.1932	0.1998	0.6070

alternative after using all three normalization methods *N*1, *N*2 and *N*3 was also exactly the same when the *MARCOS* method was applied. This is also explained by the *MARCOS* method regarding high stability when ranking the solutions [12].

– Upon using the *N*4 and *N*5 data normalization method application, the best and worst solutions were the same when the two different weighting methods were applied. However, this result is not consistent with the case of *N*1, *N*2, *N*3, and the *WSM*, *WPM,* and *TOPSIS* methods.

Table 8. Some parameters under *MARCOS* method and rank of alternatives.

Trial.	K_i^-	K_i^+	$f(K_i^+)$	$f(K_i^-)$	$f(k_i)$	Rank
A_1	0.00839	0.00895	0.51634	0.48366	0.00577	9
A_2	0.01259	0.01344	0.51634	0.48366	0.00867	8
A_3	0.02046	0.02184	0.51634	0.48366	0.01408	5
A_4	0.01820	0.01943	0.51634	0.48366	0.01253	6
A_5	0.02855	0.03048	0.51634	0.48366	0.01965	3
A_6	0.01663	0.01775	0.51634	0.48366	0.01144	7
A_7	0.03527	0.03765	0.51634	0.48366	0.02427	2
A_8	0.02120	0.02264	0.51634	0.48366	0.01459	4
A_9	0.03920	0.04185	0.51634	0.48366	0.02698	1

Table 9. Alternative rank of EN91 steel turning process.

	Normalization method										Rank [28]		
Trial.	*N*1	*N*2	*N*3	*N*4	*N*5	*N*1	*N*2	*N*3	*N*4	*N*5			
	Entropy weight					MEREC weight					*WSM*	*WPM*	*TOPSIS*
A_1	9	9	9	9	5	9	9	9	9	5	9	9	9
A_2	8	8	8	5	8	8	8	8	5	8	5	6	8
A_3	5	6	7	1	9	4	6	6	1	9	2	2	4
A_4	6	5	5	4	3	6	5	5	4	3	8	8	7
A_5	3	3	3	7	2	3	3	3	7	2	3	3	2
A_6	7	7	6	6	6	7	7	7	6	6	4	4	6
A_7	2	2	2	3	4	2	2	2	3	4	6	5	3
A_8	4	4	4	2	1	5	4	4	2	1	7	7	5
A_9	1	1	1	8	7	1	1	1	8	7	1	1	1

– In the use of *N*4, A_3 is determined to be the best alternative. However, the data in Table 4 showed that this is not true since *Ra* at A_3 is larger than *Ra* at A_9, *Rz* in A_3 is larger than *Rz* at A_9, and *MRR* at A_3 is smaller than *MRR* at A_9. Thus, concluding that A_3 to be the best is erroneous. That confirms *N*4 is not suitable to blend with *MARCOS*. Similarly, in the use of *N*5, A_8 is defined as the best alternative. However, *Ra* at A_8 is larger than at A_7, *Rz* at A_8 is also larger than at A_7, and the *MRR* at A_8 is smaller than at A_7. Hence, concluding that A_8 to be the best is erroneous. That confirms *N*5 is not suitable to blend with *MARCOS* as well.

4.2 Multi-criteria decision making of milling operation

Table 10 shows the experimental results of milling Ti-6Al-4V Titanium Alloy [31]. In that study, nine experiments were carried out, the velocity speed (v_c), feed rate (f_z) and depth of cut (a_p) were changed at each experiment. *Ra* and *MRR* were selected as two parameters of the machining process and they were also identified in each experiment. The purpose of multi-criteria decision making is to find out one of nine experiments that simultaneously obtain minimum *Ra* and maximum *MRR*. Implementation is the same as in Section 4.1, the ranked results of the solutions are presented in Table 11. Some authors [31] ranked the solutions by the *TOPSIS* method, that results were also included in this table.

The data in Table 11 revealed that: When all the five data normalization methods *N*1, *N*2, *N*3, *N*4 and *N*5 were applied, the best alternatives were the same, and also the same as the *TOPSIS* method. It can be said that the combination of *MARCOS* with all the five data normalization methods achieves positive results. The high stability for ranking the alternatives by *MARCOS* is expressed again in the fact that the 2nd and 3rd rank as normalizing with *N*1, *N*2, *N*3, *N*4, and *N*5 were similar, and the same as the *TOPSIS* method as well. Nonetheless, it should be noted that this is only correct in this case. In order to reach a conclusion, it is essential to consider more specific problems. In which, one problem was considered in Section 4.1 and the other is considered in Section 4.3.

4.3 Multi-criteria decision making of grinding operation

Experimental data of the 65G steel grinding process were used in a study [32], as shown in Table 12. In that study,

Table 10. Data of milling Ti-6Al-4V Titanium alloy experiments [31].

Trial.	Input parameter			Response	
	v_c (m/min)	f_z (mm/tooth)	a_p (mm)	Ra (μm)	MRR (cm^3/min)
A_1	60	0.03	0.2	0.281	5.42
A_2	60	0.065	0.4	0.337	1.08
A_3	60	0.1	0.6	0.737	16.25
A_4	90	0.03	0.4	0.328	21.67
A_5	90	0.065	0.6	0.321	10.83
A_6	90	0.1	0.2	0.507	2.17
A_7	120	0.03	0.6	0.359	32.5
A_8	120	0.065	0.2	0.412	43.33
A_9	120	0.1	0.4	0.636	16.25

Table 11. Solutions rank of Ti-6Al-4V Titanium alloy milling process.

Trial.	Normalization method										Rank [31]
	*N*1	*N*2	*N*3	*N*4	*N*5	*N*1	*N*2	*N*3	*N*4	*N*5	
	Entropy weight					MEREC weight					*TOPSIS*
A_1	6	5	7	7	7	7	7	7	7	7	7
A_2	8	7	8	8	9	8	8	8	8	9	9
A_3	7	8	6	5	4	5	6	5	5	4	4
A_4	3	3	3	3	3	3	3	3	3	3	3
A_5	4	4	4	6	6	6	4	6	6	6	6
A_6	9	9	9	9	8	9	9	9	9	8	8
A_7	2	2	2	2	2	2	2	2	2	2	2
A_8	1	1	1	1	1	1	1	1	1	1	1
A_9	5	6	5	4	5	4	5	4	4	5	5

twenty-seven experiments were carried out, the spindle speed (n), feed rate (f_w), depth of cut (a_r), dressing feed rate (f_d), and dressing depth (a_d) were changed at each experiment. *Ra* and *MRR* were selected as two parameters of the grinding process and they were also identified in each experiment. The purpose of multi-criteria decision making is to find out one of twenty-seven experiments that simultaneously have minimum *Ra* and maximum *MRR*. Implementation is the same as in Section 4.1, the rank results of the alternatives are presented in Table 13. The rank results of the alternatives under the *PIV* and *WASPAS* approaches are shown in this table [32].

The data in Table 13 revealed that:

- When *N*1, *N*2, *N*3 and *N*4 were used: The ranks from No. 1 to No. 9 are the same. The ranks from No. 18 to No. 27 are the same as well. Therefore, the stability in ranking the alternatives by *MARCOS* is confirmed again. In particular, these ranks coincide with the rank results using the *PIV* and *WAPSAS* methods. This gives a certain confidence in the ranked results and also confirms that the methods *N*1, *N*2, *N*3 and *N*4 combined with *MARCOS* give the high accuracy ranking results.
- In the use of *N*5, A_{27} is determined to be the best alternative. The data in Table 12 clearly disagreed with this. Specifically, in the three alternatives A_{25}, A_{26} and A_{27}, *MRR* was 101.737 (mm^3/min), but *Ra* in A_{27} is larger than in A_{25} and A_{26}. Thereby, the mix of *N*5 with *MARCOS* does not lead to the desired accuracy.

In summary, in the example 1, *N*1, *N*2, *N*3 are found suitable for combining with *MARCOS*, while *N*4 and *N*5 are not suitable for that ones. In example 2, mixing *N*1, *N*2, *N*3, *N*4 and *N*5 with *MARCOS* is equally effective. In example 3, *N*1, *N*2, *N*3, *N*4 blended with *MARCOS* gives high accuracy, while combining *N*5 with *MARCOS* is not suitable. It can be said that in all three examples implemented, only *N*1, *N*2 and *N*3 seem to be always suitable to combine with *MARCOS*. Also, the alternative ranked second is consistently determined. This shows that, the *MARCOS* method not only shows its advantages such as the ranked results of the solutions less dependent on the determining method of the weights for the criteria, and the ranked results were not reversed (as confirmed in previous studies), but also has the combining ability with the many data normalization methods. This is the outstanding advantage of the *MARCOS* method in comparing with other *MCDM* methods.

Table 12. Experimental data of 65G steel grinding process [32].

Trial.	Input parameter					Response	
	n (rev/min)	f_w (mm/rev)	a_r (mm)	f_d (mm/min)	a_d (mm)	R_a (µm)	MRR (mm^3/min)
A_1	400	0.05	0.01	100	0.005	0.295	18.843
A_2	400	0.05	0.01	100	0.01	0.332	18.843
A_3	400	0.05	0.01	100	0.015	0.370	18.843
A_4	400	0.075	0.015	150	0.005	0.399	42.39
A_5	400	0.075	0.015	150	0.01	0.436	42.39
A_6	400	0.075	0.015	150	0.015	0.474	42.39
A_7	400	0.09	0.02	200	0.005	0.489	67.813
A_8	400	0.09	0.02	200	0.01	0.527	67.813
A_9	400	0.09	0.02	200	0.015	0.564	67.813
A_{10}	600	0.05	0.015	200	0.005	0.425	42.39
A_{11}	600	0.05	0.015	200	0.01	0.460	42.39
A_{12}	600	0.05	0.015	200	0.015	0.495	42.39
A_{13}	600	0.075	0.02	100	0.005	0.312	84.766
A_{14}	600	0.075	0.02	100	0.01	0.347	84.766
A_{15}	600	0.075	0.02	100	0.015	0.382	84.766
A_{16}	600	0.09	0.01	150	0.005	0.408	50.877
A_{17}	600	0.09	0.01	150	0.01	0.443	50.877
A_{18}	600	0.09	0.01	150	0.015	0.478	50.877
A_{19}	800	0.05	0.02	150	0.005	0.417	75.348
A_{20}	800	0.05	0.02	150	0.01	0.457	75.348
A_{21}	800	0.05	0.02	150	0.015	0.497	75.348
A_{22}	800	0.075	0.01	200	0.005	0.542	56.53
A_{23}	800	0.075	0.01	200	0.01	0.582	56.53
A_{24}	800	0.075	0.01	200	0.015	0.622	56.53
A_{25}	800	0.09	0.015	100	0.005	0.398	101.737
A_{26}	800	0.09	0.015	100	0.01	0.438	101.737
A_{27}	800	0.09	0.015	100	0.015	0.478	101.737

5 Proposing data normalization methods

In Section 4, $N1$, $N2$ and $N3$ are all suitable to mix with $MARCOS$ in the multi-criteria decision problem. At this point, the author of this study wondered why the data normalization by the mean of these three methods was not used. The concept of "mean" is used commonly in statistics. There are three types of mean: the arithmetic mean, the geometric mean, and the harmonic mean. Assuming there are m values including n_1, n_2, ..., n_m, the formula for calculating of the mean of these three types is presented in (21), (22), and (23).

$$\overline{n} = \frac{1}{m}(n_1 + n_2 + \ldots + n_m) \quad \text{Calculate the arithmetic mean} \tag{21}$$

$$n = (n_1 \cdot n_2 \cdot \ldots \cdot n_m)^{\frac{1}{m}} \quad \text{Calculate the geometric mean} \tag{22}$$

$$\overline{n} = \frac{m}{\frac{1}{n_1} + \frac{1}{n_2} + \ldots + \frac{1}{n_m}} \quad \text{Calculate the harmonic} \tag{23}$$

The observation of formula (23) shows that the harmonic mean is not suitable for calculating the normalized value, in fact, there are n_i to be 0. Hence, two methods of calculating the normalized values including the arithmetic and geometric mean are used in this study. Thereby, two new methods are proposed in this paper for normalizing data based on the method VI and VII as below. Where $n_{ij}^{(1C)}$; $n_{ij}^{(1B)}$ are the normalized values based on $N1$, $n_{ij}^{(2C)}$; $n_{ij}^{(2B)}$ are the normalized values based on $N2$, and $n_{ij}^{(3C)}$; $n_{ij}^{(3B)}$ are the normalized values based on $N3$.

Method VI ($N6$)

$$n_{ij}^{(6C)} = \frac{1}{3}\left(n_{ij}^{(1C)} + n_{ij}^{(2C)} + n_{ij}^{(3C)}\right) \quad \text{if} \quad j \in C \tag{24}$$

$$n_{ij}^{(6B)} = \frac{1}{3}\left(n_{ij}^{(1B)} + n_{ij}^{(2B)} + n_{ij}^{(3B)}\right) \quad \text{if} \quad j \in B \tag{25}$$

Table 13. Alternative rank of 65G steel grinding process.

Trial.	Normalization method										Rank [32]	
	*N*1	*N*2	*N*3	*N*4	*N*5	*N*1	*N*2	*N*3	*N*4	*N*5		
	Entropy weight					*MEREC* weight					*PIV*	*WASPAS*
A_1	25	25	25	25	27	25	25	25	25	27	25	25
A_2	26	26	26	26	26	26	26	26	26	26	26	26
A_3	27	27	27	27	25	27	27	27	27	25	27	27
A_4	19	19	19	14	24	19	19	19	19	24	19	19
A_5	21	21	21	21	22	21	21	21	21	22	21	21
A_6	23	23	23	23	20	23	23	23	23	20	23	23
A_7	10	10	10	11	12	10	10	10	10	12	10	10
A_8	11	11	11	12	11	11	11	11	11	11	11	11
A_9	12	12	12	13	10	12	12	12	12	10	12	12
A_{10}	20	20	20	20	23	20	20	20	20	23	20	20
A_{11}	22	22	22	22	21	22	22	22	22	21	22	22
A_{12}	24	24	24	24	19	24	24	24	24	19	24	24
A_{13}	4	4	4	4	6	4	4	4	4	6	4	4
A_{14}	5	5	5	5	5	5	5	5	5	5	5	5
A_{15}	6	6	6	6	4	6	6	6	6	4	6	6
A_{16}	16	14	15	17	18	15	13	13	14	18	16	13
A_{17}	17	16	17	18	17	17	14	15	17	17	17	16
A_{18}	18	18	18	19	16	18	16	17	18	16	18	18
A_{19}	7	7	7	7	9	7	7	7	7	9	7	7
A_{20}	8	8	8	8	8	8	8	8	8	8	8	8
A_{21}	9	9	9	9	7	9	9	9	9	7	9	9
A_{22}	13	13	13	10	15	13	15	14	13	15	13	14
A_{23}	14	15	14	15	14	14	17	16	15	14	14	15
A_{24}	15	17	16	16	13	16	18	18	16	13	15	17
A_{25}	1	1	1	1	3	1	1	1	1	3	1	1
A_{26}	2	2	2	2	2	2	2	2	2	2	2	2
A_{27}	3	3	3	3	1	3	3	3	3	1	3	3

Table 14. Summary of alternative rank of EN91 steel turning process.

Trial.	Normalization method										Rank [28]		
	*N*1	*N*2	*N*3	*N*6	*N*7	*N*1	*N*2	*N*3	*N*6	*N*7			
	Entropy weight					MEREC weight					*WSM*	*WPM*	*TOPSIS*
A_1	9	9	9	9	9	9	9	9	9	9	9	9	9
A_2	8	8	8	8	8	8	8	8	8	8	5	6	8
A_3	5	6	7	6	7	4	6	6	4	5	2	2	4
A_4	6	5	5	5	5	6	5	5	7	7	8	8	7
A_5	3	3	3	3	3	3	3	3	3	3	3	3	2
A_6	7	7	6	7	6	7	7	7	6	6	4	4	6
A_7	2	2	2	2	2	2	2	2	2	2	6	5	3
A_8	4	4	4	4	4	5	4	4	5	4	7	7	5
A_9	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 15. Summary of alternative rank of Ti-6Al-4V Titanium Alloy milling process.

Trial.	Normalization method										Rank [31]
	*N*1	*N*2	*N*3	*N*6	*N*7	*N*1	*N*2	*N*3	*N*6	*N*7	
	Entropy weight					MEREC weight					*TOPSIS*
A_1	6	5	7	6	5	7	7	7	7	6	7
A_2	8	7	8	8	7	8	8	8	8	8	9
A_3	7	8	6	7	8	5	6	5	6	7	4
A_4	3	3	3	3	3	3	3	3	3	3	3
A_5	4	4	4	4	4	6	4	6	5	4	6
A_6	9	9	9	9	9	9	9	9	9	9	8
A_7	2	2	2	2	2	2	2	2	2	2	2
A_8	1	1	1	1	1	1	1	1	1	1	1
A_9	5	6	5	5	6	4	5	4	4	5	5

Table 16. Summary of alternative rank of 65G steel grinding process.

Trial.	Normalization method										Rank [32]	
	*N*1	*N*2	*N*3	*N*6	*N*7	*N*1	*N*2	*N*3	*N*6	*N*7		
	Entropy weight					MEREC weight					*PIV*	*WASPAS*
A_1	25	25	25	25	25	25	25	25	25	25	25	25
A_2	26	26	26	26	26	26	26	26	26	26	26	26
A_3	27	27	27	27	27	27	27	27	27	27	27	27
A_4	19	19	19	19	18	19	19	19	19	17	19	19
A_5	21	21	21	21	20	21	21	21	21	21	21	21
A_6	23	23	23	23	23	23	23	23	23	22	23	23
A_7	10	10	10	10	10	10	10	10	10	10	10	10
A_8	11	11	11	11	11	11	11	11	11	11	11	11
A_9	12	12	12	12	12	12	12	12	12	12	12	12
A_{10}	20	20	20	20	19	20	20	20	20	19	20	20
A_{11}	22	22	22	22	21	22	22	22	22	21	22	22
A_{12}	24	24	24	24	24	24	24	24	24	23	24	24
A_{13}	4	4	4	4	4	4	4	4	4	4	4	4
A_{14}	5	5	5	5	5	5	5	5	5	5	5	5
A_{15}	6	6	6	6	6	6	6	6	6	6	6	6
A_{16}	16	14	15	15	13	15	13	13	13	13	16	13
A_{17}	17	16	17	17	15	17	14	15	15	14	17	16
A_{18}	18	18	18	18	17	18	16	17	17	16	18	18
A_{19}	7	7	7	7	7	7	7	7	7	7	7	7
A_{20}	8	8	8	8	8	8	8	8	8	8	8	8
A_{21}	9	9	9	9	9	9	9	9	9	9	9	9
A_{22}	13	13	13	13	14	13	15	14	14	15	13	14
A_{23}	14	15	14	14	16	14	17	16	16	18	14	15
A_{24}	15	17	16	16	22	16	18	18	18	24	15	17
A_{25}	1	1	1	1	1	1	1	1	1	1	1	1
A_{26}	2	2	2	2	2	2	2	2	2	2	2	2
A_{27}	3	3	3	3	3	3	3	3	3	3	3	3

Method VII ($N7$)

$$n_{ij}^{(7C)} = \left[n_{ij}^{(1C)} \cdot n_{ij}^{(2C)} \cdot n_{ij}^{(3C)}\right]^{1/3} \quad \text{if} \quad j \in C \tag{26}$$

$$n_{ij}^{(7B)} = \left[n_{ij}^{(1B)} \cdot n_{ij}^{(2B)} \cdot n_{ij}^{(3B)}\right]^{1/3} \quad \text{if} \quad j \in B. \tag{27}$$

These two data normalization methods were applied to rank the alternatives in the problems in Sections 4.1–4.3. Each data normalization method is also combined with *MARCOS*, while the weights of the criteria are determined using both methods (Entropy and *MEREC*). The ranked results of the solutions using $N6$ and $N7$ data normalization methods applied for the three problems of turning, milling, and grinding process are presented in Tables 14–16, respectively. So as for the convenience of comparing, the results of ranking alternatives using $N1$, $N2$, $N3$ and the other multi-criteria decision-making methods are included in these tables.

The data in Table 14 revealed that: the best and worst alternatives using $N6$ and $N7$ are the same as $N1$, $N2$ and $N3$. Furthermore, the best and worst solutions using $N6$ and $N7$ are the same as using *WSM*, *WPM,* and *TOPSIS* methods in making decisions. That confirms $N6$ and $N7$ are suitable to blend with *MARCOS*. In addition, the stability in indicating the second and third ranked solutions was also found to be consistent when all the five data normalization methods are used.

The data in Table 15 revealed that: the first, second and third rank in the use of $N6$ and $N7$ are the same as $N1$, $N2$, $N3$, and also *TOPSIS* methods. Therefore, it can be concluded that combining $N6$, $N7$ with *MARCOS* gives accuracy in multi-criteria decision making.

The data in Table 16 revealed that: The ranks from 1 to 13, and ranks from 20 to 27 in the case of using all the five data normalization methods ($N1$, $N2$, $N3$, $N6$, $N7$) are the same and coincide in the situation of applying the *PIV* and *WASPAS* methods. This result also leads to a conclusion that $N6$ and $N7$ can be combined with *MARCOS* for high accuracy.

The above analysis demonstrates that:

- Two data normalization methods proposed in this study ($N6$ and $N7$) seem suitable to combine with *MARCO*S in multi-criteria decision making. Of the seven data normalization methods used in this study, only $N4$ and $N5$ are probably inappropriate to mix with *MARCOS*.
- In addition to identifying the best alternative, the second ranked alternative is also the same in the use of all five data normalization methods ($N1$, $N2$, $N3$, $N6$, $N7$). Consequently, the decision makers are able to consider another alternative in the case that the best approach cannot be applied for some reason (similar to the one example mentioned in the introduction). The ability to combine with all five data normalization methods further demonstrates the applicability of the *MARCOS* method in comparing to other decision-making methods. That advantage was clearly demonstrated for the used first two data normalization methods in this study ($N6$ and $N7$).

6 Conclusion

When using for multi-criteria decision making, the *MARCOS* method confirmed the outstanding advantages such as the ranked results of the solutions were very stable, these results less dependented on the weighted values of the criteria, and the reversion phenomenon of the solution rankings did not occur. However, if only using the $N1$ data normalization method (the method was used by the author who proposed the *MARCOS* method), when encountering situations such as the existence of $x_{ij} = 0$ or $\max(x_{ij}) = 0$, method $N1$ cannot be used, then obviously, the *MARCOS* method also cannot be used. In order to take the above advantage of the *MARCOS* method, this study was performed to investigate the suitability of other data normalization methods when combined with the *MARCOS* method. The results are as follows:

- Three data normalization methods including $N1$, $N2$, and $N3$ were suitable to combine with the *MARCOS* method. The two data normalization methods that were proposed in this study ($N6$ and $N7$) were also suitable to combine with the *MARCOS* method. It can be said that this is the outstanding advantage of the *MARCOS* method in comparing to other methods. In contrast, the combination of $N4$ and $N5$ with *MARCOS* should be avoided.
- With all the five data normalization methods ($N1$, $N2$, $N3$, $N6$, and $N7$), when the *MARCOS* method is used, the best and worst alternatives are determined regardless of the weighting method, data normalization method, number of solutions, number of criteria. Besides, in the mix of *MARCOS* and the data normalization methods ($N1$, $N2$, $N3$, $N6$, and $N7$), not only the best and worst solutions are defined, but also the second ranked option is consistently identified. This helps the decision makers consider another alternative in the case that the best approach cannot be used for some reason. As a result, this study recommends the use of *MARCOS* in combination with the data normalization methods ($N1$, $N2$, $N3$, $N6$, and $N7$) in multi-criteria decision making.
- In the case that $N1$ cannot be used (for example, exists $x_{ij} = 0$ or $\max(x_{ij}) = 0$), then $N2$ and $N3$ could be used to normalize the data and still having high confidence in the accuracy of the ranked results of the solutions.
- This study affirmed that $N6$ and $N7$ are fairly suitable to use with *MARCOS* for multi-criteria decision making. However, more research is needed to figure out if there are similar results from the combination of $N6$, $N7$ with other multi-criteria decision-making methods (*TOPSIS*, *VIKOR*, etc.).
- Up to now, data normalization by any method would not have been performed if existing at least one qualitative criterion. So, improving the *MARCOS* method or finding other methods to rank the solutions in this case is the content that needs to be conducted in the next studies. On the other hand, the combination of the *MARCOS* method with the data normalization methods $N1$, $N2$, $N3$, $N6$ and $N7$ has only been evaluated through three examples in the field of mechanical processing. Whether that combination is successful or not when applied in other areas such as logistics, information technology, strategic options, etc. also need to be investigated further.

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